Statistics
Spring 2023
Lecture 11

Feb 19-8:47 AM

Class QZ 4

Consider a binomial prob. dist. with $n=250$ & $p=.6$

1) Find $\mu = np = 250(.6) = 150$ ✓

2) Find $P(x=165) = \text{binompdf}(250,.6,165) = .008$ (n p x) ✓

3) Find $P(x \le 170) = \text{binomcdf}(250,.6,170) = .996$ (n p x) ✓

Apr 18-9:09 PM

Ages of Pilots are normally dist. with $\mu = 46$ Yrs and $\sigma = 8$ Yrs.

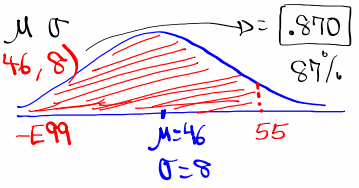
If we randomly select one pilot, find the Prob. that his/her age (x) is below 55 Yrs. $x < 55$

$P(x < 55)$

$=$ normalcdf((-) E99, 55, 46, 8)

L U μ σ

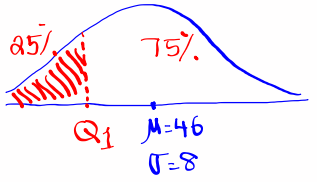
2nd [ ]



Find Q_1, Rounded to a whole, for ages of Pilots.

$x = Q_1 =$ invNorm(.25, 46, 8)

$= 40.604 \approx \boxed{41}$



Apr 25-6:52 PM

Clear all lists.

Store 1, 3, 5, 7, and 9 in L1.

Use [1-Var Stats] with L1 only to find

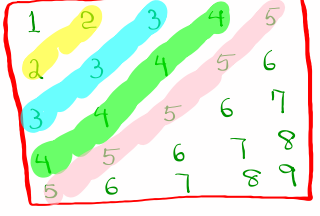
$\mu = 5$ $\sigma = 2.828$ $\sigma^2 = 8$

Now take all Samples of [Size 2] with replacement from this data.

1,1	1,3	1,5	1,7	1,9
3,1	3,3	3,5	3,7	3,9
5,1	5,3	5,5	5,7	5,9
7,1	7,3	7,5	7,7	7,9
9,1	9,3	9,5	9,7	9,9

25 Samples of Size 2

Now find $\bar{x}$ of each Sample

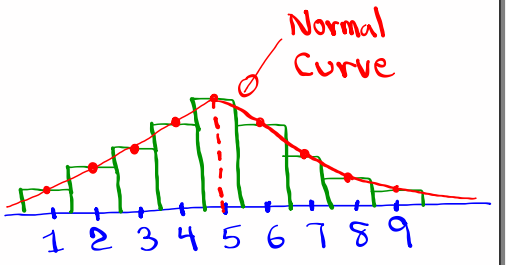


25 means

Apr 25-7:00 PM

$\bar{x}$	$P(\bar{x})$
1	1/25
2	2/25
3	3/25
4	4/25
5	5/25
6	4/25
7	3/25
8	2/25
9	1/25

Draw Prob. dist. histogram using $\bar{x}$ & $P(\bar{x})$



$\bar{x} \rightarrow L2$, $P(\bar{x}) \rightarrow L3$

Use [1-Var stats] with L2 & L3 to find

$\mu_{\bar{x}} = 5$ $\quad \sigma = 2$ $\quad \sigma_{\bar{x}}^2 = 4 = \frac{8}{2} \leftarrow \sigma^2 \leftarrow n$

Apr 25-7:07 PM

Central – Limit Theorem

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Suppose we have normal dist with $\mu = 5800$ and $\sigma = 400$ and we take Sample of Size 25,

$\mu_{\bar{x}} = \mu = 5800$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{400}{\sqrt{25}} = \frac{400}{5} = 80$

Apr 25-7:15 PM

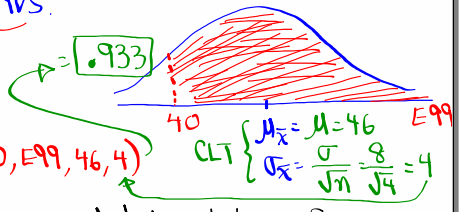
Ages of Pilots are normally dist with $\mu = 46$ and $\sigma = 8$.

If we randomly Select 4 pilots, $n = 4$, find the prob. that their mean age $\bar{x}$ is above 40 Yrs.

$\bar{x} > 40$

$P(\bar{x} > 40)$

= normalcdf(40, E99, 46, 4)



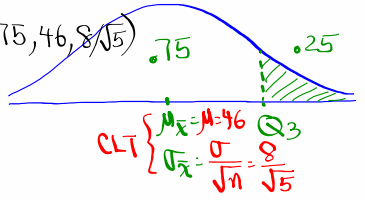
= .933

Find $\bar{x} = Q_3$, Rounded to whole #, for randomly Selected group of 5 pilots.

$\bar{x} = Q_3 = \text{invNorm}(.75, 46, 8/\sqrt{5})$

= 48.413

≈ 48



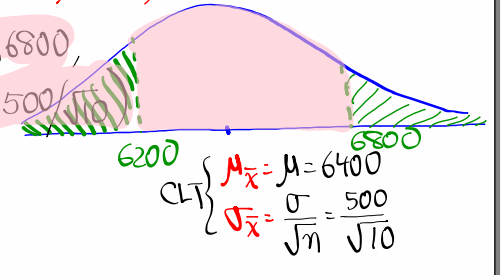
Apr 25-7:20 PM

Salaries of nurses are normally dist with the mean of \$6400/mo. and Standard deviation of \$500/mo. N(6400, 500)

If we randomly Select group of 10 nurses, $n = 10$, find the Prob. that their mean $\bar{x}$ Salary is below \$6200 or above \$6800.

$P(\bar{x} < 6200 \text{ OR } \bar{x} > 6800)$

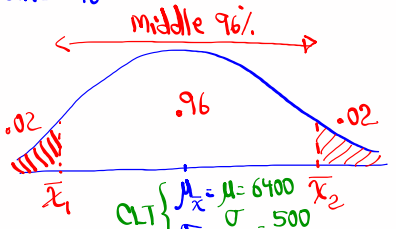
= 1 − normalcdf(6200, 6800, 6400, 500/√10)



= .109

Apr 25-7:29 PM

For randomly selected group of 15 nurses, find two mean salaries that separate the middle 96% from the rest. Round up your ans to whole #.



$1 - .96 = .04$

$.04 \div 2 = .02$

CLT: $\mu_{\bar{x}} = \mu = 6400$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{500}{\sqrt{15}}$

$\bar{x}_1 = \text{invNorm}(.02, 6400, 500/\sqrt{15})$

$= 6134.862 \approx \boxed{6135}$ $\approx \boxed{6666}$

$\bar{x}_2 = \text{invNorm}(.98, 6400, 500/\sqrt{15}) = 6665.138$

Exam II: Next week

SG 1 to SG 21

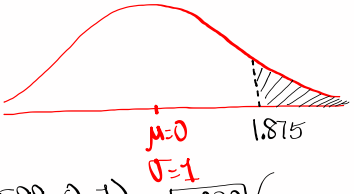
Lecture, followed by exam.

Apr 25-7:37 PM

Find the area to the right of

$Z = 1.875.$

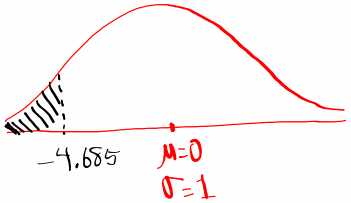
$N(0,1)$



normalcdf(1.875, E99, 0, 1) = .030 ✓

Find twice the area to the left of

$Z = -4.685.$



2*normalcdf(-E99, -4.685, 0, 1)

$= \boxed{2.803 \times 10^{-6}}$

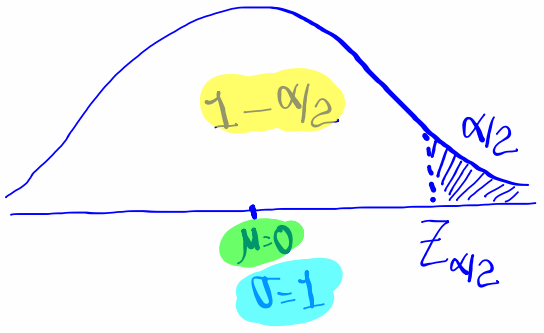
Apr 25-7:55 PM

$Z_{\alpha/2}$ is the value that separates right area of $\alpha/2$ from the rest.

α ← Alpha

↑ Significance level

$0 < \alpha < 1$



to find $Z_{\alpha/2}$

use invNorm(Left Area, μ, σ)

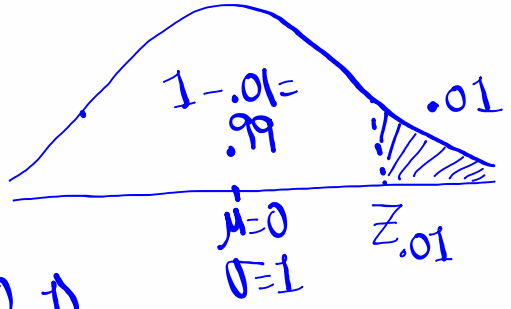
Apr 25-8:01 PM

Find $Z_{.01}$

Right Area

$\alpha/2 = .01$

$\alpha = .02$



$Z_{.01}$ = invNorm(.99, 0, 1)

$\approx \boxed{2.326}$

Apr 25-8:04 PM

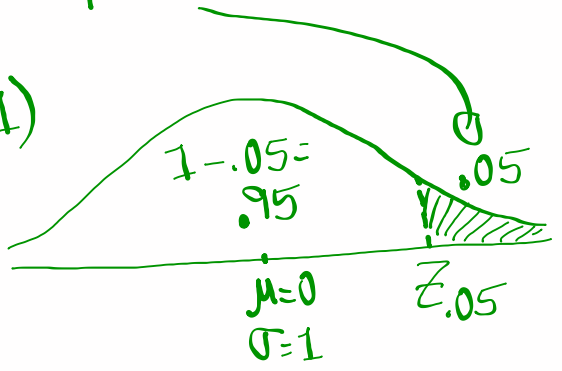
Find $Z_{\alpha/2}$ for $\alpha = .1$

$\alpha/2 = .1/2 = .05$

Find $Z_{.05}$ ← Right Area

$Z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$



Apr 25-8:07 PM

α ← Alpha ← Significance level

$0 < \alpha < 1$

$(1-\alpha)\cdot 100\%$ ← Confidence level

middle area

Find $Z_{\alpha/2}$ for 98% Conf. level.

Middle Area = .98

98% middle

$1 - .98 = .02$

$.02 \div 2 = .01$

$Z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

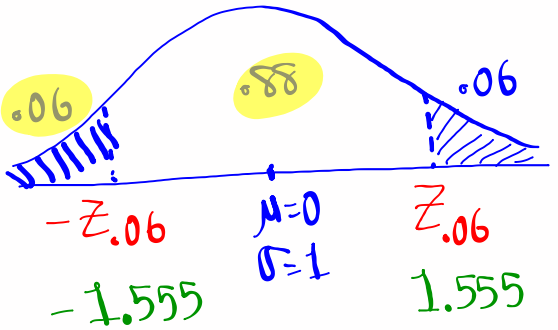
Apr 25-8:09 PM

Find $\pm z_{\alpha/2}$ for 88% Conf. level.

Middle Area = .88

$1 - .88 = .12 \leftarrow \alpha$

$.12 \div 2 = .06 \leftarrow \alpha/2$



$z_{.06} = \text{invNorm}(.94, 0, 1)$

$= 1.555$

Apr 25-8:14 PM

t-Dist.

1) Graph is symmetric, bell-shape, Total Area = 1

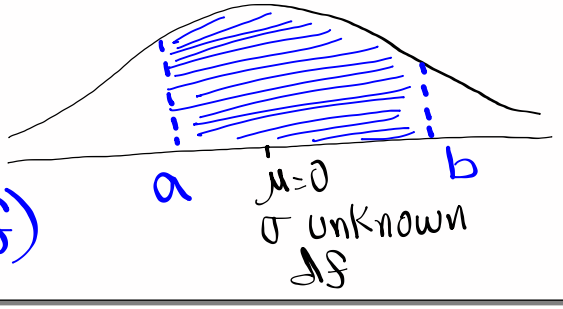
2) $\mu = 0$, σ unknown

3) It comes with degrees of freedom

$P(a < t < b)$

2nd VARS tcdf

(Lower, Upper, df)

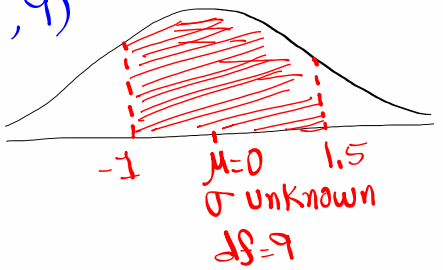


Apr 25-8:17 PM

Find $P(-1 < t < 1.5)$ with $df = 9$.

$= tcdf(-1, 1.5, 9)$

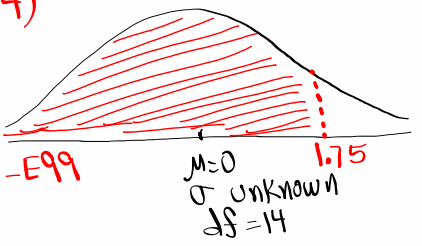
$= .744$



Find $P(t < 1.75)$ with $df = 14$

$= tcdf(-E99, 1.75, 14)$

$= .949$

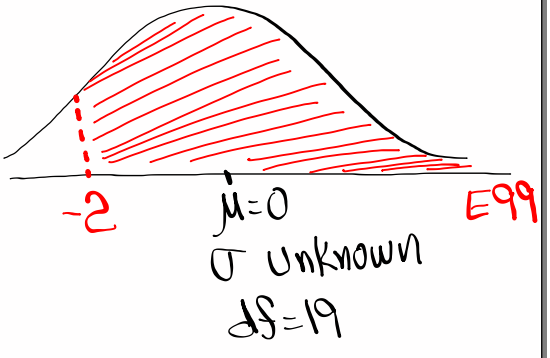


Apr 25-8:20 PM

Find $P(t > -2)$ with $df = 19$.

$= tcdf(-2, E99, 19)$

$= .970$



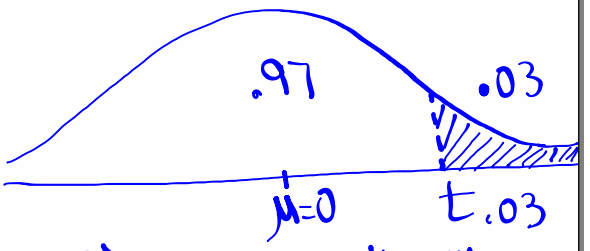
Apr 25-8:25 PM

Find $t_{.03}$ with $df = 12$.

$t_{.03}$: Right Area

$\frac{\alpha}{2} = .03$

$\alpha = .06$



$t_{.03} = invT(\underbrace{\text{Left Area}}_{.97}, \underbrace{df}_{12})$

$= \boxed{2.076}$

σ unknown

$df = 12$

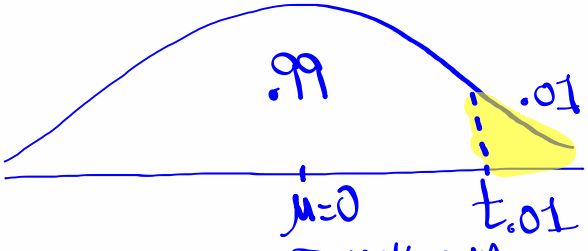
Use App Wabbit EMU

Apr 25-8:28 PM

Find $t_{\alpha/2}$ for $\alpha = .02$ with $df = 10$.

$\alpha/2 = .02/2 = .01$

$t_{.01}$



$t_{.01} = invT(.99, 10) = \boxed{2.764}$

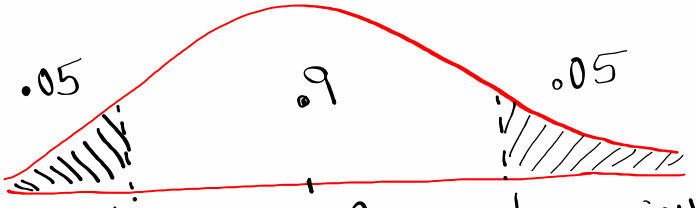
σ unknown

$df = 10$

Apr 25-8:33 PM

Find $\pm t_{\alpha/2}$ for 90% Conf. level with df=19.

middle Area .9



Apr 25-8:36 PM

χ^2 - Dist.

↖ chi-Square

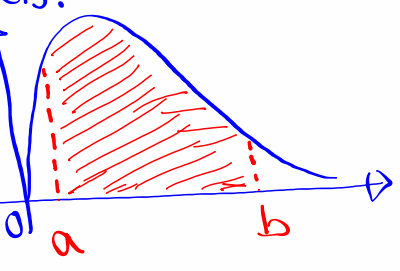
1) Graph begins at 0, and Skewed to the right.

2) Not Symmetric, Not bell-shape but total area = 1

3) It comes with df.

$P(a<\chi^2<b)$

χ^2cdf(Lower, Upper, df)

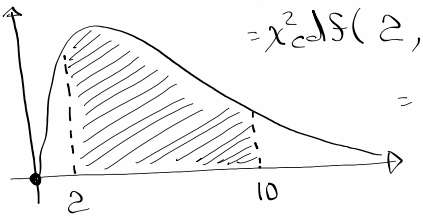


Apr 25-8:40 PM

Find $P(2 < x^2 < 10)$ with $df = 8$.

$= x^2cdf(2, 10, 8)$

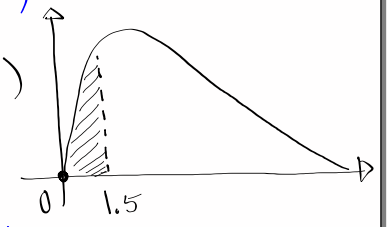
$= .716$



Find $P(x^2 < 1.5)$ with $df = 5$.

$= x^2cdf(0, 1.5, 5)$

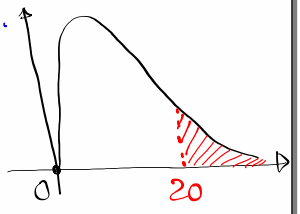
$= .087$



$P(x^2 > 20)$ with $df = 6$.

$= x^2cdf(20, E99, 6)$

$= .003$

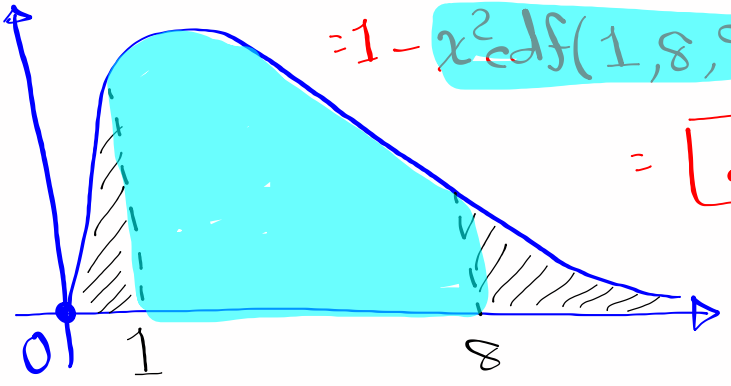


Apr 25-8:44 PM

Find $P(x^2 < 1 \text{ OR } x^2 > 8)$ with $df = 9$.

$= 1 - x^2cdf(1, 8, 9)$

$= .535$



Apr 25-8:51 PM

F-Dist.

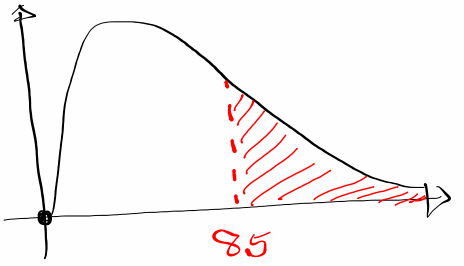
It is similar to χ^2-Dist but comes with two df.

Ndf Numerator df

Ddf Denominator df

use fcdf(L, U, Ndf, Ddf)

find P(F>8.5) with Ndf=5, Ddf=20.



fcdf(8.5, E99, 5, 20)

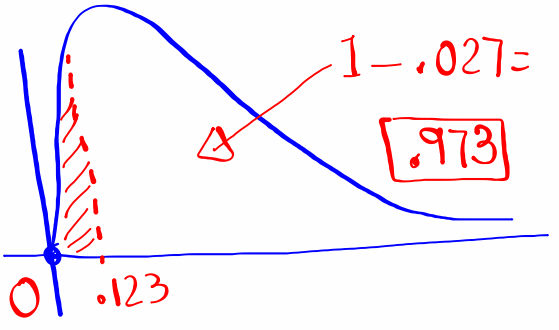
$= 1.908 \times 10^{-4}$

Apr 25-8:55 PM

find P(F<.123) with Ndf=4, Ddf=24

= fcdf(0, .123, 4, 24)

= .027



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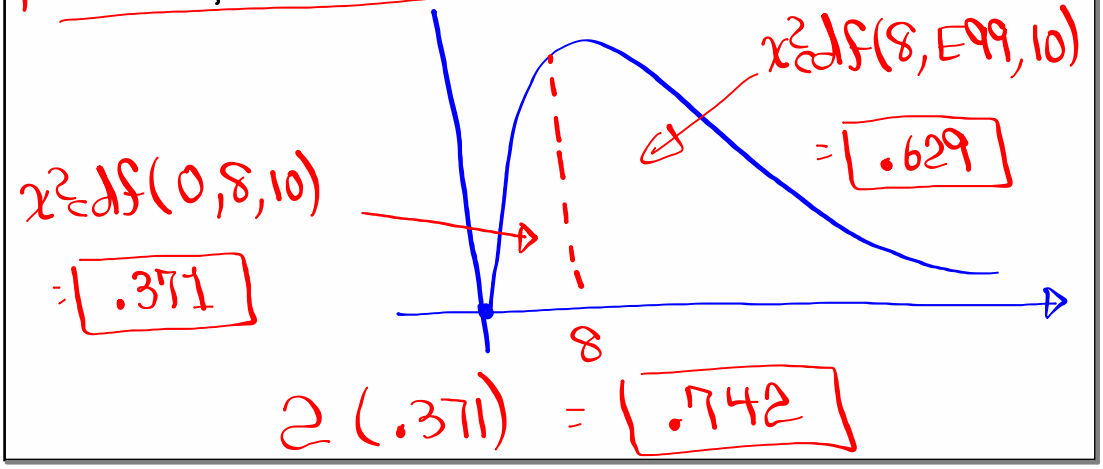
Given $\chi^2 = 8$, $dF = 10$

Find the area on each side,

multiply the Smaller area by 2.

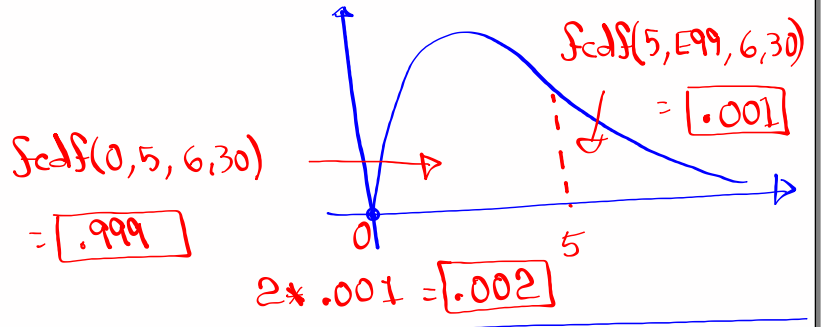


Apr 25-9:05 PM

Given $F = 5$, $Ndf = 6$, $Ddf = 30$

Find the area on each side,

multiply the Smaller area by 2.



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